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A Program for Computing Reflection Coefficients

IRVIN KAY and SIDNEY LIEBERMAN

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Irvin Kay and Sidney Lieberman

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C. 2

Abstract

We consider an electromagnetic wave incident on a stack of dielectric slabs with different dielectric constants. It is desired that the reflection coefficient at the front of the structure be computed. In this paper a program for Univac, which is used to obtain the desired reflection coefficient, is discussed. In particular the way in which the information is put into Univac and the form in which the answers appear is discussed.

Table of Contents

I. Introduction

The program, for Univac, is designed to compute the reflection coefficient for a plane wave incident from the left at any angle on a stack of dielectric slabs. The dielectric constants, permeability and conductivity of these slabs, as well as of the medium in which the wave has been traveling are given data. In general the program will compute the reflection coefficient both for a wave which is polarized normal to the plane of incidence and for one polarized parallel to the plane of incidence. The program will, in either case, give both the real and imaginary part of the reflection coefficient at the front of the stack.

The program computes the reflection coefficient for different angles of incidence and the same stack of slabs in the following manner. First it calculates the real and imaginary parts of the reflection coefficients for normal and parallel polarization for normal incidence, i.e. at an angle of incidence of zero radians. It then computes the same quantities for an angle of incidence $\Delta\theta$ radians. ($\Delta\theta$ is typed directly into Univac by the operator--see Section 2). It then automatically computes the same quantities for an angle of incidence $2(\Delta\theta)$, for $3(\Delta\theta)$ etc. It proceeds to do this n times until it has computed the reflection coefficient for an angle of incidence $(n-1)\Delta\theta$. (n, the number of angles for which the program computes the reflection coefficient, is also typed directly into Univac (See Section 2.).)

The program is a fixed point program, so that appropriate scaling must be done to all the input data as described in Section 3. However, since the reflection coefficient is always less than or equal to one in absolute value, the answers will appear with no scaling necessary.

In the exceptional case when the reflection coefficient across an interior interface for a critical angle of incidence is larger than one in absolute value, Univac may overflow and stop.

2. Physical Operation

The operation of the program requires five magnetic tapes. The actual program is placed on tape one. Tape two has a temporary blank on which the answers are stored before they are edited (see Section 4). Tape three contains the data for the problem, e.g., the number of slabs, the dielectric constants and width of each slab, to be put on the tape as described in Section 3. Tape four has the edit routine , and tape five is a printer tape upon which the final edited answers appear.

When all tapes are mounted, the operator Initial-Reads tape one. The program proceeds from there until the typwriter at the side of Univac types out ANG INC = , at which point the operator types in the desired angle increment ($\Delta\theta$ in radians); i.e., if one wished to compute the reflection coefficient for angles which differ by .076 radians, one would then type in 007 600 000 000. After this " type-in", the program proceeds to the next " type-out" NO ANG =, at which point the operator types in the desired number of angles as though the decimal point appeared to the extreme right; i.e., if one wished to compute the reflection coefficient of a given stack of dielectric slabs for thirty-two angles at increments of .05 radians, one would type in after ANG INC = , 000 500 000 000, and after NO ANG = , 000 000 000 032.*

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* During this process the dial on the typewriter next to Univac should be kept on " normal" as opposed to " computer digit."

Univac now proceeds to compute all the reflection coefficients for the different angles of incidence. When it completes the computation, the edit routine automatically takes over. This routine continues until it types out DATE at which point the operator types in the date. When the machine finishes editing, it types out ANYMORE at which point the operator types in " printer break point" followed by eleven " ignores" . Univac then types out the number of blocks appearing on tape five and stops.

3. Data Input

The following data are necessary for the operation of the program: the number of dielectric slabs, the dielectric constants, and the thickness of each slab. If we assume that the stack of dielectric slabs begins at the origin, we can consider the end points of the slabs, which determine their thickness, to be known. The coordinates of the end points are, in fact, the data to be put in.

We put the number of slabs in location 000 of the first block of tape three as though the decimal appeared to the extreme right; i.e., if we had twelve slabs, location 000 would contain 000 000 000 012. In location 001 we have the " word" . ? 0 000 000 004 where ? is to be replaced by the appropriate scaling factor described at the end of this section. In location 002, 003, 004, 005, we place, respectively, the real and imaginary parts of the reflection coefficients for normal and parallel polarization at the extreme right of the stack of slabs. For example, if we had a perfect reflector to the right of our stack of slabs we would have -1, 0, -1, 0 in 002, 003, 004, 005 respectively, using, instead of -1, the approximation -.99 999 999 999.

Now into 006 007 008 go the ε_o μ_o σ_o of the medium to the left of the stack of slabs all scaled by the same power of 10 so that the greatest of

them will be less than 1. In 009 we place a representation of the scaling factor of the $\varepsilon_o \mu_o \sigma_o$ which is minus the power of 10 used, and we put it in the eighth digit. For example, if we scale $\varepsilon_o \mu_o \sigma_o$ by 10^{-2}, 009 contains 000 000 020 000. In 010 we place the coordinate of the end point of the slab, but since we are ssuming that the stack begins at the origin, 010 contains 0. We then continue with ε, μ, σ, in 011, 012, 013--all scaled by a power of 10 with minus the power of 10 again appearing in the eighth digit of 014. In 015 we put z , the coordinate of the end point of slab one.

It is clear that scaling for the z's may eventually become necessary. We will then be required to find a common scaling factor for all z's again represented as a power of 10. Minus this power of 10 is then to be put in the second digit of 001, which also contains some permanent symbols. For example, if the greatest z to be scaled is z = 23.7, all z's are to be scaled by 10^{-2}, and 001 contains .20 000,000 004. It is important to remember that no matter what the scaling factor is for the z's, all symbols in 001 remain unchanged except that the scaling factor is placed in the second digit.* (See Appendix I for sample input data.)

4. Data Output

When Univac stops, the edited answers appear on tape five. When tape five is put on the printer the following tabs are to be set. Margin 6, T - 1 = 10, T - 2 = 15, T - 3 = 20 , T - 4 = 15, T - 5 = 9, in which case the answers appear as follows:

* One should never use a scaling factor smaller than 10^{-9} anywhere in the program since the result would then lose all significant digits.

Date	Refl Coefficient			
	Normal Pol		Parallel Pol	
Angle	RE R	IM R	RE R	IM R

with the appropriate reflection coefficient and angle appearing in the appropriate column. (See Appendix II for sample output corresponding to sample input of Appendix I.)

If one sets the comma break point before Initial Reading one, it is possible to stop the program, if desired, before it reaches the edit routine. In this case the real and imaginary parts of the reflection coefficient for normal and parallel polarization, and for normal incidence, appear in locations 940, 941, 942, 943 of Univac, respectively. This device is very useful and timesaving when one wishes to see only a few unedited answers and perhaps change certain data.

5. Units and Maximum Capacity

The data to be put on tape three (Section 3.) is described in M.K.S. units. However, relative units may be used if the following corrections are made.

If relative units are used, then ε is to be replaced by ε' where $\varepsilon' = \frac{\varepsilon}{\varepsilon^*}$, and ε^* is the ε of free space in M.K.S. units. σ is to be replaced by $\sigma/\omega\varepsilon^*$ where ω is the angular frequency. z is to be replaced by $2\pi f_o/c_o$ z where f_o, c_o are the frequency and velocity of light in free space, measured in units corresponding to those used for z.

The scaling factors are put in exactly as before except that this time they are chosen to scale the new data. The reflection coefficient appears exactly as before.

The program is designed to compute the reflection coefficient for a

maximum of ninety-four slabs. However, this difficulty can be overcome in the following manner. Assume that we have a stack of n slabs $n > 94$. Then we compute the reflection coefficient for the following stack of slabs n, n-1, n-2,...ν where $n - \nu < 93$, remembering to put $\varepsilon_{\nu-1}$ $\mu_{\nu-1}$ $\sigma_{\nu-1}$ in the data for $\varepsilon_o \mu_o \sigma_o$. When we have computed the reflection coefficient for these $n - \nu + 1$ slabs we put the result for the real and imaginary parts and the two polarizations into 002, 003, 004, and 005, to correspond to a new set of data for slabs $\nu - 1$ ... μ. This process can be continued until the reflection coefficient at the far left for all n slabs has been computed.

The program is not designed to compute the reflection coefficient for different frequencies automatically, since this would require changing the data each time. It is planned, in the near future, to add a subroutine which would change the data for us, thus allowing computation for all frequencies as well as for all angles of incidence.

6. Example

Assume that we are given an electromagnetic wave incident on a stack of two dielectric slabs. The wave is traveling in a medium of given dielectric constants and is approaching the structure from the left. To the extreme right of the structure we have a perfect conductor. (See Figure A.)

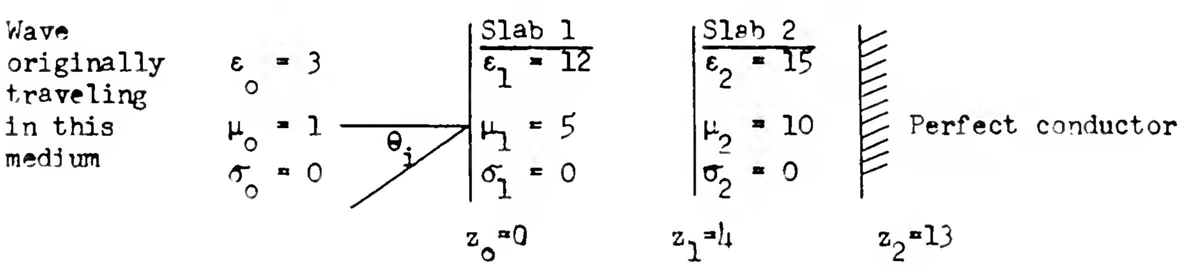

Figure A

We are given the dielectric constants of the medium in relative units as follows:

$$\varepsilon_o = 3 \qquad \varepsilon_1 = 12 \qquad \varepsilon_2 = 15$$

$$\sigma_o = 1 \qquad \mu_1 = 5 \qquad \mu_2 = 10$$

$$\mu_o = 0 \qquad \sigma_1 = 0 \qquad \sigma_2 = 0$$

$$z_o = \frac{2\pi f_o}{c_o} x_o = 0 \quad z_1 = \frac{2\pi f_o}{c_o} x_1 = 4 \qquad z_2 = \frac{2\pi f_o}{c_o} x_2 = 13,$$

where x_1 is the physical distance of the end of slab one from the origin.

We would like to know the reflection coefficient at the origin for all angles of incidence of the form $\theta_n = n\,\Delta\theta \quad n = 0 \ldots 30$ and $\Delta\theta = .05$ radians.

The data is then to be put on tape three as it appears in Appendix I. The " type ins" (See Section 3.) will then be ANG ING = 000 500 000 000 and NO ANG = 000 000 000 030.

The results then appear on tape five. A printed version of that tape appears in Appendix II.

Appendix I

0	000	000	000	002	Reflection coefficient for 2 slabs being computed	
1	.20	000	000	004	Scaling factor for z's is 10^{-2}	
2	-99	999	999	999	Real part normal pol	Perfect reflector backing up stack of slabs
3	000	000	000	000	Im part normal pol	
4	-99	999	999	999	Real part par pol	
5	000	000	000	000	Im part par pol	
6	030	000	000	000	$\varepsilon_o = 3$	
7	010	000	000	000	$\mu_o = 1$	
8	000	000	000	000	$\sigma_o = 0$	
9	000	000	010	000	Scaling factor for 0th slab is 10^{-1}	
10	000	000	000	000	Stack of slabs begin at origin $z_o = 0$	
11	012	000	000	000	$\varepsilon_1 = 12$	
12	005	000	000	000	$\mu_1 = 5$	
13	000	000	000	000	$\sigma_1 = 0$	
14	000	000	020	000	Scaling factor for 1st slab is 10^{-2}	
15	004	000	000	000	$z_1 = 4$ first slab ends 4 units from origin	

16	015	000	000	000	$\varepsilon_2 = 15$
17	010	000	000	000	$\mu_2 = 10$
18	000	000	000	000	$\sigma_2 = 0$
19	000	000	020	000	scaling factor for 2nd slab is 10^{-2}
20	013	000	000	000	$z_2 = 13$. Second slab ends 13 units from origin

Appendix II

MAY 11, 1959 REFL COEFFICIENT 001

	NORMAL POL		PARALLEL POL	
ANGLE	RE R	IM R	RE R	IM R
0.000	-97811259478	-20806221316	-97811259478	-20806221316
0.050	-95377990799	-30076873746	-97811258770	-20806223425
0.100	-82926904908	-55888809887	-97811262517	-20806273344
0.150	-48698987381	-87347850861	-97815016717	-20821147520
0.200	010969126021	-99401353921	-97806826064	-20859060036
0.250	073411635934	-67906937559	-97790010680	-20937210007
0.300	099974871005	002380799557	-97764213216	-21055935127
0.350	066135707375	075015466063	-97719040414	-21288985412
0.400	-17970753539	098376567214	-97649670959	-21601833442
0.450	-92744604585	037407850605	-97549884671	-22044102853
0.500	-72685561596	-68689755982	-97411568666	-22642327241
0.550	052718854114	-84982274897	-97228981682	-23407615693
0.600	078713807326	061681607638	-96993968165	-24355445733
0.650	-67044917695	074200790354	-96708079422	-25484670353
0.700	-88930402914	-45737819016	-96347109564	-26807123386
0.750	014977531088	-98875433399	-95909467496	-28346533054
0.800	097959991137	020136401044	-95393528638	-30025954680
0.850	-50795964666	086144628372	-94788399098	-31894606582
0.900	-90714094413	-42089206784	-94087075969	-33895519865
0.950	023388330263	-97233710700	-93303378167	-36003755321
1.000	090445213560	042667734495	-92431575707	-38174730264
1.050	-43610439794	089996038264	-91494996921	-40373740499
1.100	-98994125138	014183649878	-90502725446	-42557126903
1.150	-78945369232	-61382226309	-89468174266	-44681155051
1.200	036040283320	-93290624756	-88444278812	-46680707984
1.250	040161112217	091592476603	-87435888828	-48535126883

	NORMAL POL		PARALLEL POL	
ANGLE	RE R	IM R	RE R	IM R
1.300	-71606131183	069810576884	-86489945659	-50208060102
1.350	-94914892240	031488792527	-85640517228	-51637435028
1.400	-99400402891	010941462259	-84921583956	-52819805111
1.450	-99993254600	001160770999	-84359641504	-53708927887
1.500				

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